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Investigations in Protothetic

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In this article I present some results of five years' research into Leśniewski's protothetic.[1] I outline deductions from the axiom A_n considerably shorter than those previously known (see [9]) and I derive the laws of implication from this axiom without using the rule of extensionality.[2] Since this paper can best be read in the light of articles by Professor Sobociński published in this Journal (see [8], [9], and [10]), I have largely adopted his conventions of symbolism, and the following symbols in particular:

α The rule permitting definitions of new constants (see [8], pp. 58–59).
β The rule for distributing quantifiers (see [8], p. 59).
0 An informal abbreviation for '$[u] . u$'.
1 An informal abbreviation for '$[u] . u .\equiv. [u] . u$'.

From the axiom

$$A_n[pq] :: p \equiv q .\equiv .\cdot. [f] \therefore f(pf(p0)) .\equiv: [r] : f(qr) .\equiv. q \equiv p$$

we prove the following theorems:

D1	$[p] . p \equiv \mathrm{As}(p)$	$[\alpha]$
L1	$[fp] \therefore f(pf(p0)) .\equiv: [r] : f(\mathrm{As}(p)r) .\equiv. \mathrm{As}(p) \equiv p$	$[A_n\ q/\mathrm{As}(p);\ \mathrm{D1}]$
D2	$[pr] \therefore p \equiv r .\equiv. \mathrm{As}(p \equiv r) :\equiv. \mathrm{Vr}(pr)$	$[\alpha]$
L2	$[pr] . \mathrm{Vr}(pr)$	$[\mathrm{D2};\ \mathrm{D1}\ p/p \equiv r]$
L3	$[pr] : \mathrm{Vr}(\mathrm{As}(p)r) .\equiv. \mathrm{As}(p) \equiv p$	$[\mathrm{L1}\ f/\mathrm{Vr};\ \mathrm{L2}\ r/\mathrm{Vr}(p0)]$

We can now establish the following four metarules of procedure:

S I (see [9], pp. 114–115) *If we have in the system a thesis of the type*

$[x, y, \ldots] : A .\equiv. B$

(with or without an initial quantifier), then we can add the corresponding thesis

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