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An Intuitionistic Sheffer Function

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The purpose of this note is to present a ternary propositional function which is a Sheffer function in the Heyting propositional calculus. We shall also consider some related Sheffer functions in positive logic.

Although it is easy to guess what should be the general notion of a Sheffer function in a propositional calculus, we shall first fix our terminology. Following [2] and [1], we shall say that a set of functions F is a *Sheffer set* for a set of functions G iff every member of G can be defined by a finite number of compositions from the members of F. A set F is an *indigenous Sheffer set* for G iff F is a Sheffer set for G and G is a Sheffer set for F. A function f is an (*indigenous*) *Sheffer function* for G iff $\{f\}$ is a (indigenous) Sheffer set for G. Of course these notions will interest us here only when the functions in question are propositional functions. Unless stated otherwise, $\rightarrow$, $\wedge$, $\vee$, $\neg$, $\leftrightarrow$, $\bot$, and $\top$ will stand for the usual Heyting propositional functions.

In [1] Hendry has shown that there is no binary indigenous Sheffer function for $\{\rightarrow, \wedge, \vee, \neg\}$. In that paper it is also stated that $\{\leftrightarrow, \vee, \neg\}$ is an indigenous Sheffer set for $\{\rightarrow, \wedge, \vee, \neg\}$. More precisely, $\{\leftrightarrow, \vee\}$ is an indigenous Sheffer set for $\{\rightarrow, \wedge, \vee\}$, since in the Heyting propositional calculus we can prove

$$(A \rightarrow B) \leftrightarrow ((A \vee B) \leftrightarrow B)$$
$$(A \wedge B) \leftrightarrow ((A \vee B) \leftrightarrow (A \leftrightarrow B)) \ .$$

(A useful survey of such equivalences can be found in [3] and [4], p. 21.)

Some further economy was achieved by Schroeder-Heister in [7]. He shows that $\{s, \bot\}$ is an indigenous Sheffer set for $\{\rightarrow, \wedge, \vee, \neg\}$, where s is a ternary propositional function defined by

$$s(A_1, A_2, A_3) \leftrightarrow ((A_1 \leftrightarrow A_2) \vee A_3) \ .$$

Then in the Heyting propositional calculus we can prove

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