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New Axioms for Mereology

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In this paper I shall present several new axioms and axiom systems for mereology with an account of their origin. I shall also outline a proof that the two most interesting of these are adequate sole axioms for mereology.

Professor Lejewski discovered the first mereological axioms which employ the terms '*Kl*' and '*ov*' (see [2], p. 43, theses E2 and E4). His axioms have thirteen ontological units each, and so are longer than most of the known single axioms. I have discovered two new axioms for these terms:

2.1 $[AB] \because\because A \in ov(B) .\equiv\because\because [\exists Ca] :: C \in a :: [DE] \because\cdot [A] \therefore A \in E .\equiv: [B] : A \in ov(B) .\equiv. [\exists C] . C \in a . C \in ov(B) :: E \in E . C \in ov(D) ::\supset. A \in ov(D) . B \in ov(D)$

3.1 $[Aa] :: A \in Kl(a) .\equiv\because [\exists B] . B \in a \because [Bb] :: [A] : A \in b .\equiv. [\exists Ccd] . A \in Kl(c) . B \in Kl(d) . C \in c . C \in d \therefore Kl(a) \in Kl(a) \therefore\supset: A \in b .\equiv. [\exists C] . C \in a . C \in b.$

Axiom 2.1 is only ten ontological units long, and so is shorter than all other single axioms now known except one for the term '*ex*', which is also ten units in length (see [3], p. 66, thesis E6). Axiom 3.1 is eleven units long. Unlike the just mentioned axiom for '*ex*' and the earlier axioms for '*Kl*' and '*ov*', Theses 2.1 and 3.1 have no functor variables. To show that 2.1 and the definitions associated with it are indeed theses of mereology, I shall derive theses corresponding to them from the following axiom system for the term '*el*'.[1]

1.1 $[AB] : A \in el(B) .\supset. B \in B$

1.2 $[ABC] : A \in el(B) . B \in el(C) .\supset. A \in el(C)$

1.3 $[A] : A \in A .\supset. A \in el(A)$

1.4 $[Aa] :: A \in Kl(a) .\equiv\therefore A \in A \therefore [B] : B \in a .\supset. B \in el(A) \therefore [B] : B \in el(A) .\supset. [\exists CD] . C \in a . D \in el(B) . D \in el(C)$

1.5 $[Aa] : A \in a .\supset. Kl(a) \in Kl(a)$

1.6 $[AB] \therefore A \in ov(B) .\equiv: A \in A : [\exists C] . C \in el(A) . C \in el(B).$

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