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The Rule of Procedure *Re* in Łukasiewicz's Many-Valued Propositional Calculi

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A modified form of modus ponens is used to give new formalizations of Łukasiewicz's finite and infinite-valued propositional calculi. In the infinite-valued case, we establish the independence of the axiom schemes.

Let C and N be the primitive implication and negation functors respectively of Łukasiewicz (see [4]). All the propositional calculi we consider here have 1 as the only designated truth-value. The usual primitive rule of procedure in formalizations of propositional calculi with C and N, or with C as the only primitive functor(s) is modus ponens (with respect to C). We consider here an alternative rule of procedure which occurred as a derived rule in [7], p. 101, and which has been considered in [1] and [2] as a primitive rule of procedure for the two-valued propositional calculus. This rule of procedure is as follows.

Re Let P, Q and R be formulas and let the result of replacing one occurrence of the subformula CPQ in R by Q be S. Then, if P and R are correct formulas, S is a correct formula.

Clearly, modus ponens is a special case of *Re*. Reductions in the number of axiom schemes of a similar nature to those we obtain here have been described in the two-valued case in [11].

We shall give several formalizations of the $\aleph_0$-valued and m-valued propositional calculi with C and N as the only primitive functors and *Re* as the only primitive rule of procedure. We also give several formalizations of the $\aleph_0$-valued and m-valued propositional calculi with C as the only primitive functor and *Re* as the only primitive rule of procedure. For each formalization we shall establish weak deductive completeness (i.e., the provability of all generalized tautologies), and for the formalizations of the $\aleph_0$-valued propositional calculi we establish the independence of the axiom schemes and primitive rule of procedure.

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