Notre Dame Journal of Formal Logic
Volume 26, Number 4, October 1985

Recursively Saturated ω_1-like Models of Arithmetic

ROMAN KOSSAK*

Two models of PA are called *similar* if they are elementarily equivalent and have the same standard systems. In [3] it was shown than any two recursively saturated ω_1-like similar models are $L_{\infty\omega_1}$-equivalent, and that there is at least a continuum of pairwise nonisomorphic, recursively saturated, ω_1-like models which are similar to a given countable recursively saturated model of PA. In this paper we show that the number of models with the above properties is in fact $2^{\aleph_1}$, and we may also construct them to be mutually not elementarily embeddable.

Thus, it is natural to ask in what extensions of $L_{\omega\omega}$ it is possible to describe recursively saturated ω_1-like models up to isomorphism. Since we have $2^{\aleph_1}$ pairwise nonisomorphic models, countable languages are out of the consideration (at least when $2^{\aleph_0} < 2^{\aleph_1}$). This applies in particular to the stationary logic $L(aa)$. In Section 3 we take a look at finitely determinate structures, which were studied by Eklof and Mekler in [1]. The reason is that the proof of our theorem on the existence of $2^{\aleph_1}$ pairwise nonisomorphic models (Theorem 2.4) does not exclude the possibility that a stationary logic version of the isomorphism theorem is true for finitely determinate models. Theorem 3.5 shows that this is not the case. We still may have $2^{\aleph_1}$ pairwise nonisomorphic, recursively saturated, ω_1-like finitely determinate models which have the same standard systems and satisfy the same $L(aa)$ theories. Moreover, from a lemma due to Shelah, it follows that the models constructed are also $L_{\infty\omega_1}(aa)$-equivalent. The proof of Theorem 3.5 uses the $\diamondsuit$ principle and the existence of Kurepa trees.

No knowledge of stationary logic, except for the Eklof-Mekler characterization of $L(aa)$-equivalence for finitely determinate structures and/or the Shelah lemma, is needed for our considerations. In fact, all of our results about

*The results of this paper were obtained in early spring of 1983, when the author was at Bedford College in London under visiting fellow research grant GR/C/30672 from the Science and Engineering Research Council. He would like to thank Wilfrid Hodges for the invitation, great hospitality, and valuable conversations and advice on the subject of the paper.

Received June 17, 1983; revised October 29, 1984