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Definable Partitions and Reflection Properties for Regular Cardinals

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The purpose of the present paper is to study the relation between definable partitions and reflection properties of regular cardinals. It turns out that in contrast to Σ_1^1 reflection, which does not lead to a large cardinal axiom (see Section 2), Π_1^1 reflection, which is studied in association with definable stationary subsets of κ (see Section 3) and definable partition properties (see Section 4), leads to a large cardinal axiom. In particular it follows (see Section 4) that the least regular uncountable cardinal which satisfies a certain partition relation lies strictly between the first uncountable inaccessible and the first uncountable Mahlo cardinal (assuming the axiom of constructibility $V = L$).

1 Introduction and preliminaries The Jensen hierarchy $(J_\alpha : \alpha \in \mathrm{Ord})$ of constructible sets is defined in [2]. L is the universe of constructible sets. Only structures of the form $\mathbf{M} = (M, \in, R_1, \ldots, R_r)$ will be considered, where M is a nonempty set and $R_1, \ldots, R_r$ are relations on M. The Levy hierarchies Σ_n, Π_n of formulas in the language with predicate symbols $\in, S_1, \ldots, S_n$ (the arity of each S_i is the same as the arity of R_i), and the corresponding sets of $\Sigma_n(\mathbf{M})$, $\Pi_n(\mathbf{M})$, $\Delta_n(\mathbf{M})$ of relations on the set M, are defined as usual (see [2]). A formula ϕ is a first-order formula if it is in Σ_n, for some $n \geq 0$. The set of first-order formulas is denoted by Σ_ω. Any formula of the form $\exists V_1 \ldots \exists V_m \phi$, $\forall V_1 \ldots \forall V_m \phi$, where the formula $\phi = \phi(V_1, \ldots, V_m, x_1, \ldots, x_k)$ is first order, $V_1, \ldots, V_m$ are second-order variables, $x_1, \ldots, x_k$ are first-order variables, is respectively called Σ_1^1, Π_1^1.

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