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Two λ-Extensions of the Theory of Homogeneous Simple Types as a Second-Order Logic

NINO B. COCCHIARELLA

In the theory of simple logical types as originally conceived, it is meaningless for one predicate expression to occur in one of the subject or argument positions of another unless the latter is assigned a higher logical type than the former *within the grammar of the object language*; and therefore it is meaningless in particular for any predicate expression to apply to itself, i.e., to occur in one of its own subject positions. Russell's paradox of predication is thereby avoided, of course, but the price is high, for the resulting theory is not an accurate representation of the role of predicates in natural language where predicate expressions can apply not only to the nominalizations of other predicates but to their own nominalizations as well—and without regard at all for the notion of a logical type.

In the theory of logical types as a second-order logic, on the other hand, predicate expressions are typed within the grammar of the object language only in the way they are typed in standard second-order logic, i.e., only with respect to their degree or number of subject positions (adicity), and they are allowed otherwise to meaningfully occur in the subject or argument positions of other predicates, and of themselves as well, without regard to the notion of a logical type. Russell's paradox of predication can be avoided, it turns out, not by resorting to the notion of a logical type as a part of the grammar of the object language but rather only as a part of the metalinguistic description of the conditions under which properties and relations are to be posited by means of the grammar of the object language. The difference is crucial, needless to say, since it allows for a more accurate representation of the role of predicates and predication in natural language. The resulting theory is not, to be sure, a secondorder logic in the "standard" sense used today (though it does contain the latter), but it is a second-order logic in the traditional or pre-type-theoretical sense in which quantifier expressions are allowed to reach into both subject and

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