Notre Dame Journal of Formal Logic
Volume 26, Number 3, July 1985

Incompactness in Regular Cardinals

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Annotated Contents

Introduction: We review the old axioms, the theorems on λ singular and two examples.

Section 1 A general theorem on lifting incompactness: Continuing [6], we give additional axioms axiomatizing "free amalgamation", and prove with them transfer theorems of the form: "if there is a λ-free not λ^+-free pair A/B, $|A| = \lambda$ then there is a μ-free not μ^+-free pair A'/B', $|A'| = \mu$".

Section 2 Particular incompactness theorems: We apply Section 1 for some examples, and show an almost equivalence between a colouring number problem of graphs and a combinatorial problem $IC(\lambda, \delta)$ [existence of pairwise disjoint end segments of branches of a tree].

Section 3 Canonical counterexamples for $PT(\lambda, \kappa^+)$: We define a λ-set for λ an uncountable regular cardinal, which is a kind of $(<\omega)$-dimensional stationary set. Using this we analyze counterexamples to $PT(\lambda, \kappa^+)$. As a consequence we prove:

If $PT(\lambda, \aleph_1)$ fails, then there are countable $A_i (i < \lambda)$ such that $\{A_i: i < \lambda\}$ has no transversal but for every $\alpha < \lambda$ we can find $\beta_i (i < i(*))$ $\alpha = \{\beta_i: i < i(*)\}$ and $A_{\beta_i} - \bigcup_{j<i} A_{\beta_j}$ is infinite.

Section 4 Some investigation of PT: We prove $PT(\lambda, \kappa^+) \equiv PT(\lambda, \kappa^{++})$ and characterize the λ for which $PT(\lambda, \lambda)$.

*This research was partially supported by the United States–Israel Binational Science Foundation, the N.S.F., and the Canadian Research Council.

Received December 11, 1981; revised March 14, 1984