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An Exposition of Shelah's 'Main Gap': Counting Uncountable Models of ω-Stable and Superstable Theories

L. HARRINGTON and M. MAKKAI*

Introduction Throughout this introduction, we let T be a countable complete theory in (finitary) first-order logic having infinite models.

Two of the outstanding conjectures of model theory concern the number $I(T,\kappa)$ of isomorphism types of models of T having a fixed cardinality κ. Morley's conjecture says that $I(T,\kappa)$ is a monotonically increasing function of κ, for uncountable cardinals κ: κ uncountable, and $\kappa < \lambda$ imply $I(T,\kappa) \leq I(T,\lambda)$. The other is Vaught's conjecture: $I(T,\aleph_0) \leq \aleph_0$, or $I(T,\aleph_0) = 2^{\aleph_0}$.

Saharon Shelah's deep and extensive work in the exploration and classification of all possible complete theories can be seen as motivated to a large extent by Morley's conjecture. The results of this work point toward the possibility that Morley's conjecture will eventually be proved by giving more or less explicitly all possible spectrum-functions $\kappa(\geq \aleph_1) \mapsto I(T, \kappa)$, with each possibility (hopefully) conforming to Morley's requirement.

In [4], Shelah proved that $I(T,\kappa) = 2^\kappa$ $(\kappa \geq \aleph_1)$ for all unstable T. One of the main results of [5] is that the same holds for all T that are not superstable. At the end of [5], some partial results are given for very special totally

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