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From Preference to Utility: A Problem of Descriptive Set Theory

JOHN P. BURGESS*

1 Introduction Some years ago J. H. Silver proved that a co-analytic equivalence relation on a Polish space has either countably many or continuum many equivalence classes. Later L. Harrington greatly simplified the complicated original proof. The present paper is a sort of footnote to Harrington's lectures on these matters. It will be shown that information developed in his proof settles a problem of (hyper-)theoretical mathematical economics first investigated by Wesley [13] and Mauldin [8]. Namely, it will be shown that any family of closed preference orders that is parametrized in a Borel fashion can be represented by a family of continuous utility functions parametrized in an absolutely measurable fashion. Though the author is greatly indebted to Mauldin's work [8], the treatment of the problem here will be self-contained. Background and motivation for problems of this kind can be found in [6], Section 2.1. Terminology and notation pertaining to descriptive set theory will be as in [9].

2 Definitions Throughout let $\mathcal{Y}$ be a topological space. A *preference order* on $\mathcal{Y}$ is any transitive, connected binary relation $\leqslant^*$. Associated are the *strict* preference and *indifference* relations given by:

$$x <^* y \longleftrightarrow x \leqslant^* y \ \& \ {\sim} y \leqslant^* x$$
$$x \equiv^* y \longleftrightarrow x \leqslant^* y \ \& \ y \leqslant^* x.$$

Note that $\equiv^*$ is an equivalence relation, and that $\leqslant^*$ induces a linear order on its equivalence classes. $[x]^*$ will denote the equivalence class of x. $\leqslant^*$ will be

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