Notre Dame Journal of Formal Logic
Volume 26, Number 2, April 1985

Recursively Saturated Models Generated by Indiscernibles

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The theorem which is proved here has its origins in a question raised by A. Macintyre: Is there a recursively saturated model of Peano Arithmetic which is generated by a set of indiscernibles? To give this question respectability, we understand PA to be formalized so as to include terms for all definable functions. Since recursively saturated models are in some sense large whereas models generated by indiscernibles are small, the positive answer to Macintyre's question obtained by Abramson and Knight [3] was unexpected. Their proof showed that every consistent extension of PA has a countable, recursively saturated model which is generated by a set of indiscernibles. The countability comes as no surprise, for by stretching and shrinking the indiscernibles generating a recursively saturated model, one can obtain indiscernible generators for a recursively saturated model of arbitrary infinite cardinality.

This answer to Macintyre's question suggests the following modification of his question: Is *every* countable, recursively saturated model of PA generated by a set of indiscernibles? We demonstrate here that this question also has a positive answer. It is natural to consider variations of this question with PA replaced by some other theory, such as an extension of *ZFC* which has definable Skolem functions (e.g., $ZFC + V = L$). The answer to the question for such theories is also positive. What is unusual is that not all Skolem functions need be definable, but in order to carry out the proof, the existence of what may be called a β-function, which is a binary function encoding all finite sequences, is required.

*Research was supported in part by NSF Grant MCS 79-05028.

Received January 15, 1982; revised July 7, 1982