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On the Possible Number $no(M)$ = The Number of Nonisomorphic Models $L_{\infty,\lambda}$-Equivalent to M of Power λ, for λ Singular

SAHARON SHELAH*

Introduction Let M be a model of power λ, with λ relations, each with $<\lambda$ places and of power $\leq\lambda$. What can be

$$no(M) = \{N/\cong : N \equiv_{\infty,\lambda} M,\ \|N\| = \lambda\}\ ?$$

We assume $V = L$ (otherwise there are independence results (by [8])). It is known that

(A) If $cf\,\lambda = \aleph_0$, it can be only 1 (by Scott [5] for $\lambda = \aleph_0$, and generally by Chang [1], essentially).

(B) If λ is regular uncountable and not weakly compact it can be 1 or 2^λ (it can be 2^λ, see [3]; cannot be $\neq 1, 2^\lambda$: for $\lambda = \aleph_1$ by Palyutin [4], for any λ by [6]).

(C) If λ is weakly compact $>\aleph_0$ then it can be any cardinal $\leq\lambda^+$ (by [7]).

We prove here

(D) If λ is singular of uncountable cofinality, $no(M)$ can be any cardinal $\chi < \lambda$ (and also $\chi = 2^\lambda$). (This follows by 3.18 here.)

So we answer the question from [7], bottom of p. 26. The second question there, top of p. 26, is answered trivially by 1.4.

Notation: We consider functions as relations.

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