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Notre Dame Journal of Formal Logic
Volume 25, Number 4, October 1984

Individual Concepts as Propositional Variables in $ML^{\nu+1}$

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1 Introduction The modal languages ML^ν and ML_*^ν of Bressan (to be described in more detail in the second part of this introduction) are presented in [4] and [5]; substantially, ML_*^ν is obtained from ML^ν by adding propositional variables and constants. For every positive integer ν, the modal language ML^ν is based on a *type-system* τ^ν which has ν types $(1, \ldots, \nu)$ for *individual terms* and, accordingly, the semantical structures for ML^ν (the ML^ν-interpretations) are constructed starting from ν *individual domains* $D_1, \ldots, D_\nu$ and a set Γ of (elementary) *possible cases* (elsewhere called *worlds* or *points*), briefly, Γ-cases. The individual terms of type r of ML^ν are assumed to range over *individual concepts* (of type r) which are functions from Γ into D_r. This holds similarly for the ML_*^ν-interpretations, where, in addition, the propositional variables range over sets of possible cases. In every interpretation for ML^ν (or ML_*^ν) the conceivability relation between possible cases is $\Gamma \times \Gamma$ and, hence, the corresponding calculi MC^ν and MC_*^ν are based on Lewis's S5.

If we consider an $ML^{\nu+1}$-interpretation in which $D_{\nu+1}$ is a two-element set, then the individual concepts of type $\nu + 1$ can be considered as characteristic functions of subsets of Γ and hence they serve to represent propositions. In this paper this representation is used to reduce the concepts of *ML_*^ν-validity* and *general ML_*^ν-validity* (see Definition 2.2) to the analogous concepts for $ML^{\nu+1}$. In this way, the completeness of the calculus MC_*^ν (with respect to general ML_*^ν-interpretations) can be deduced from that of $MC^{\nu+1}$, which is proved in [14]. In particular, in Section 3 a correspondence between $ML^{\nu+1}$-interpretations (in which $D_{\nu+1}$ is $\{0,1\}$) and ML_*^ν-interpretations is defined, which becomes a bijection when restricted to general interpretations. In Section 4 it is proved that a formula p of ML_*^ν is valid (or valid in a general sense) iff the same holds for a suitable correspondent of it in $ML^{\nu+1}$. Furthermore, in Section 5,

*I wish to thank the referee for his detailed suggestions which have been very helpful for the second version of this paper.

Received December 22, 1982; revised November 16, 1983