Notre Dame Journal of Formal Logic
Volume 25, Number 3, July 1984

Correcting the Tableau Procedure for *S*4

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The tableau procedure for normal modal logics, as given in Kripke [3], may be summarized as follows.[1] For simplicity, we assume (when appropriate) that the tableaux in question have vacant righthand (False) columns, and that the beginning formulae are jointly satisfiable. From an initial set of wffs, a tableau is generated by standard truth-functional procedures. Then, whenever it would not be superfluous to do so, the following two rules are applied: Each possibility wff, of the form *Mp*, is Advanced to begin a new tableau stipulated to be accessible from the old one, with the operand *p* as its initial formula. Necessity wffs of the form *Lq* are Executed by placing the operand *q* in each accessible tableau, and the process is continued.

The result is an array or tree of tableaux. When the procedure runs out of things to do, the tree terminates and a Kripke model for the original formulae may be read off, taking the set of tableaux in the array as the set of "possible worlds" and the access relation *R* between tableaux as the access relation between worlds.

*S*4, which defines *R* as reflexive and transitive, raises a special problem. There are *S*4-satisfiable formulae which do not lead to termination but rather yield an infinite tree. To manage these, Kripke provides the following expedient. If a tableau is a duplicate of one earlier in the construction, it is not subjected to the Advancement procedure. It thereby blocks that particular path through the tree. When all paths are blocked, the tree terminates.

Such a "blocked" tree will not yield a model by the same recipe used for other modal systems. Instead, Kripke gives us the following modification. First, assemble all duplicate tableaux into equivalence classes *H*, and let the set of *H*'s represent the "possible worlds". Second, generate a derivative relation $\mathcal{R}$ among the *H*'s, using the relation *R* among the tableaux, by the recipe:

> $H_n \ \mathcal{R}\ H_m$ iff there are tableaux t_n and t_m in H_n and H_m, respectively, such that $t_n\ R\ t_m$.

Received April 15, 1983; revised June 13, 1983