Notre Dame Journal of Formal Logic
Volume 25, Number 3, July 1984

Trees and Finite Satisfiability: Proof of a Conjecture of Burgess

GEORGE BOOLOS*

The method of trees, expounded in such books as Jeffrey's [2] and Smullyan's [3], provides a sound and complete positive algorithmic test for unsatisfiability (and hence, for validity as well).[1] On occasion, the method can also be used to demonstrate finite satisfiability, i.e., truth in some model with a finite domain. Applied to the sentence $\exists xFx$, for example, the method yields the one-branch tree:

$$\begin{array}{c} \exists xFx \\ | \\ Fa \end{array} ,$$

from which the argument of the usual completeness proof proves the existence of a model with a one-element domain in which the sentences $\exists xFx$ and Fa are both true. But the method does not invariably demonstrate the finite satisfiability of a finitely satisfiable sentence. $\forall x\exists yRxy$, for example, is true in every one-element model in which R is interpreted as the identity relation. Applied to this sentence, the method produces the infinite one-branch tree:

*I am grateful to John Burgess, Hugues Leblanc, and Perry Smith for helpful correspondence. The content of the present paper was (re-)discovered by Smith soon after its main result was found. I am also grateful to R. C. Jeffrey.

Received June 13, 1983