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Epistemic Semantics for Classical and Intuitionistic Logic

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1 In this paper I propose to explain how several nonstandard semantic systems for the propositional logic can be seen as employing essentially the same principles for defining the notion of logical entailment and the more basic idea of assignment of truth-values to molecular sentences. The systems are Kleene's three-valued matrix for the strong connectives, Łukasiewicz's three-valued matrix, supervaluations, and Beth's semantics for intuitionistic logic. Traditionally these systems have been viewed as dividing into two quite different approaches to semantic theory depending on the philosophical interpretation of the truth-values which they employ. Classical bivalent semantics, supervaluations, Łukasiewicz's three-valued logic, and frequently Kleene's strong connectives are presented as varieties of what Dummett calls realism, the view that truth-values and their informal readings are properly analyzed in terms of a correspondence theory of truth in which sentences are understood to describe a real world. Intuitionism and the strong connectives as Kleene originally interprets them represent varieties of antirealism or epistemic semantics.[1] Intuitionistic semantics is bivalent; an assignment of T represents the epistemic fact that a sentence is provable and one of F the fact that it is not provable. Kleene represents essentially the same constructivist notion of acceptability, but he uses the narrower concept of effectively decidable—in his words, "decidable by the algorithms"—and elects to represent the possibilities in terms of three values: a sentence is assigned T if it is effectively decidable, F if its negation is, and N if neither it nor its negation is. (In the presentation below Beth's bivalent semantics are recast similarly into a trivially equivalent three-valued format.)

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