The Fundamental S-Theorem—
A Corollary

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For reasons set out in [4] and [5], it is of immense significance that the system S (for *syllogism*) satisfies the following condition:[1]

Powers property   *For every formula A of S, A → A is unprovable in S.*

It is of equal and indeed equivalent significance that the system P-W (for *pure* ticket implication *minus waffly* axioms) satisfies the following condition:

Belnap property   *For every pair A,B of distinct formulas of P-W, at least one of A→B, B→A is a nontheorem of P-W.*

Put contrapositively, what the Belnap property says is that if A and B are *provably equivalent* in P-W, then A is the very same well-formed formula as B.

That S and P-W had their corresponding properties was a long-standing—and recalcitrant—conjecture in the area of relevant logic. Building on work of Belnap, Powers, Dwyer, and Meyer, Martin eventually found a (surprisingly difficult) proof, which Martin and Meyer recount in [4]. As for the significance

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