

The Fundamental S-Theorem— A Corollary

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For reasons set out in [4] and [5], it is of immense significance that the system S (for *sylogism*) satisfies the following condition:¹

Powers property *For every formula A of S , $A \rightarrow A$ is unprovable in S .*

It is of equal and indeed equivalent significance that the system $P-W$ (for *pure* ticket implication *minus waffly* axioms) satisfies the following condition:

Belnap property *For every pair A, B of distinct formulas of $P-W$, at least one of $A \rightarrow B, B \rightarrow A$ is a nontheorem of $P-W$.*

Put contrapositively, what the Belnap property says is that if A and B are *provably equivalent* in $P-W$, then A is the very same well-formed formula as B .

That S and $P-W$ had their corresponding properties was a long-standing—and recalcitrant—conjecture in the area of relevant logic. Building on work of Belnap, Powers, Dwyer, and Meyer, Martin eventually found a (surprisingly difficult) proof, which Martin and Meyer recount in [4]. As for the significance

*This paper grew out of joint work by Martin and Meyer. Dwyer, with whom we have lost touch, may be surprised to find himself an author. Accordingly, he is not to be held responsible for any of our editorial comments—or, for that matter, for the details of the arguments set out. But key steps in those arguments do rest upon his work. In addition to those already mentioned, we are also indebted, in particular, to P. Thistlewaite, in the course of conversation with whom the truth of the Main Lemma became apparent, and to S. Giambrone, for help in preparing the manuscript.