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Separating Minimal, Intuitionist, and Classical Logic

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Classical, two-valued propositional logic contains intuitionist logic. Intuitionist logic in turn contains minimal logic. Standard formulations of the classical system, however, tend to make it difficult to determine whether a given classical thesis is purely classical, is classical and intuitionist, or belongs in all three systems.

The present paper offers formulations of classical implication-negation logic that make separation of its intuitionist and minimal components very easy. Section 1 deals with some preliminaries. Section 2 gives a classical axiom base with no dependent axioms, which has proper subaxiomatics giving intuitionist and minimal logic. The final section offers a natural deduction style counterpart of the axiomatic system.

1 ***Preliminaries*** Our point of departure is the standard intuitionist axiomatic used by Horn in [2]. Since Horn proves that this base has the separation property, it is clear that the axioms in implication and negation are sufficient for all intuitionist theses in these connectives. There are four such axioms: *CpCqp, CCpCqrCCpqCpr, CCpNqCqNp,* and *CNpCpq.* The rules of inference are modus ponens and substitution for variables. A minimal logic base is obtained from this intuitionist one simply by omitting the axiom *CNpCpq.*

For present purposes, rather than take implication and negation as primitive, it is better to take implication and a constant proposition 0, and define negation. This can be done because the minimal *C–N* system given by

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