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The Simple Consistency of a Set Theory Based on the Logic CSQ

ROSS T. BRADY*

This paper proves the simple consistency of the set theory *CST*. *CST* has the Generalized Comprehension Axiom (*GCA*), $(\exists y)(\forall x)(x \in y \longleftrightarrow A)$, and the Extensionality Rule, $x = y \Rightarrow x \in w \longleftrightarrow y \in w$, where $x = y =_{df} (\forall z)(z \in x \longleftrightarrow z \in y)$. *CST* is based on a logic *CSQ*, which is semantically described below.

CSQ Primitives

1. $\sim, \&, \rightarrow, \forall$ (connectives and quantifier)
2. $f, g, h, f', \ldots$ (predicate constants)
3. $x, y, z, x', \ldots$ (individual variables)
4. $a_1, a_2, a_3, a_4, \ldots$ (individual constants).

CSQ Formulas

1. An individual variable or constant is a term.
2. If $t_1, \ldots, t_n$ are terms and f is a predicate constant, then $ft_1 \ldots t_n$ is an atomic formula.
3. If A and B are formulas and x is an individual variable then $\sim A$, $A \& B$, $A \rightarrow B$ and $(\forall x)A$ are formulas.

A sentence is a formula with no free variables.

A *CSQ model structure* (*CSQ m.s.*) consists of ordered triples $\langle T, K, R\rangle$, such that K is a set, T is a member of K, and R is a two-place relation on K, with the following postulates holding: For $\alpha \in K$,

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