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Submodels and Definable Points in Models of Peano Arithmetic

ŽARKO MIJAJLOVIĆ*

1 Introduction In this paper we consider some definable sets and elements in countable nonstandard models of Peano arithmetic (abbreviated by P). Definable elements and their properties were considered by Jensen and Ehrenfeucht [5] and McAloon [7]. We investigate other properties of these points, and relate them to intersections of submodels of countable nonstandard models of formal arithmetic. When in this paper we speak of nonstandard models of Peano arithmetic we assume that they are countable.

We now introduce some terminology and notation. By L_P we denote the language of P. By $\underline{M}$, $\underline{N}$, . . . we denote models of L_P or simple expansions of this language, and by M, N, . . . we denote their domains respectively. The $\underline{\omega}$ stands for the standard system of natural numbers. We shall abbreviate $a_0, \ldots, a_n \in M$ by $\vec{a} \in M$. If $a \in M$ then $\underline{a}$ denotes the name of a.

As usual, by $M \subseteq_e N$ ($M <_e N$, $M \subseteq_c N$, $M <_c N$) we denote respectively that N is an end extension (elementary end extension, cofinal extension, elementary cofinal extension) of M.

Let Γ be a set of formulas of a language L, and let $\underline{A}$, $\underline{B}$ be some models of L. A formula ϕ of L is a Γ-formula if $\phi \in \Gamma$. Assume $\underline{A} \subseteq \underline{B}$. Then $\underline{A} \subseteq_\Gamma \underline{B}$ iff for all Γ-formulas ϕ of L and all $\vec{a} \in A$, $\underline{A} \models \phi\vec{a}$ implies $\underline{B} \models \phi\vec{a}$. We write $\underline{A} <_\Gamma \underline{B}$ if "implies" is replaced by "iff" above. An element $a \in A$ is a Γ-element (in $\underline{A}$) iff a is defined by a Γ-formula in $\underline{A}$. In the case of P this is equivalent to $\underline{M} \models a = \mu x \phi x$, $\phi x \in \Gamma$. The set $T \cap \Gamma$ is sometimes denoted by T_Γ.

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*I presented some of my early results at the Logic Conference in Marseille, 1981 (Corollary 2.9.1). There I had a short but inspiring discussion on these matters with D. Marker, who informed me of a generalization belonging to him and A. Wilke (Corollary 2.7.2).

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