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The Class of Neat-Reducts of Cylindric Algebras is Not a Variety But is Closed w.r.t. HP

ISTVAN NEMETI

Problem 2.11 formulated on p. 464 of [11] asks whether the class $\mathbf{Nr}_\alpha CA_\beta$ of all α-dimensional neat-reducts of β-dimensional cylindric algebras is closed under the formation of homomorphic images and subalgebras or not (for ordinals $\alpha < \beta$).

Theorem 1 below formulates the answer for every pair $\alpha < \beta$ of ordinals. At the end of this note some corollaries are formulated. Finally, a conjecture is stated which seems to be provable by the constructions used in the proofs of Theorem 1 here and II.8.6 in [12], p. 266.

Independently of us, Roger Maddux [13]-[14] obtained a partial solution of Problem 2.11 of [11] proving that $\mathbf{S}\,\mathbf{Nr}_3 CA_\beta \neq \mathbf{Nr}_3 CA_\beta$ if $\beta \geqslant 5$.

We shall use the notations of the monograph [11]; e.g., 1.5.8 refers to "item" 1.5.8 of [11], and p. 489 refers to page 489 of [11]. We shall often refer to items in the textbook [12] on cylindric set algebras (i.e., on representable *CA*s). Since [12] consists of two parts, we shall refer to an item n in the First Part by I.n and to an item n in the Second Part by II.n; e.g., I.1.1 is on p. 4, and II.1.1 is on p. 145.

Theorem 1 *For arbitrary ordinals $\alpha < \beta$, (i) and (ii) below hold.*

(i) $\mathbf{H}\,\mathbf{Nr}_\alpha CA_\beta = \mathbf{Nr}_\alpha CA_\beta$.

(ii) $\mathbf{S}\,\mathbf{Nr}_\alpha CA_\beta \neq \mathbf{Nr}_\alpha CA_\beta$ *if and only if* $1 < \alpha$.

Proof: In the proof we shall extensively use the notations of [11] without reference or any kind of warning; e.g., "s^i_j", "${}_\alpha s(i,j)$", "Cl_Γ". All these are collected at the end of [11] under the title "Index and symbols," p. 489.

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