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Some Preservation Results for Classical and Intuitionistic Satisfiability in Kripke Models

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Translations of classical into intuitionistic formal systems, as defined by Gödel and others (for a survey see [2], Section 81; [4] p. 41; or [6]), provide among other things a method for determining which classically valid formulas are intuitionistically valid. All of the translations share the following property: if T is an intuitionistic theory and T^c its classical counterpart (obtained, e.g., by adding the law of excluded middle) and if φ is a formula in an appropriate language (built up with connectives $\vee$, $\wedge$, $\rightarrow$, $\neg$, $\exists$, $\forall$) and φ' its translation then:

$$T^c \vdash \varphi \leftrightarrow \varphi' \text{ and}$$
$$T^c \vdash \varphi \text{ iff } T \vdash \varphi'.$$

The simplest (to describe) translation consists in attaching a double negation to each subformula. As a result we get an imbedding of the classical theory into the "negative fragment" of the intuitionistic theory that consists of formulas constructed without $\vee$ and $\exists$ from decidable (or doubly negated) atomic formulas. It may appear then as though the differences between classical and intuitionistic systems are due to disjunction and the existential quantifier. From the classical point of view, $\vee$ and $\exists$ could be regarded as new connectives (while classical disjunction is defined in terms of negation and conjunction and the existential quantifier is defined in terms of the universal quantifier). Such explanations are actually given in many popular accounts of intuitionism. Given Kripke models, however, the definition of forcing suggests that the real culprits are implication (as well as negation as its special case) and the universal quantifier. This also seems to be in better accord with the intuitionist interpretations of logical connectives. The results which will be presented here support this view.

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