

## Questions Concerning Possible Shortest Single Axioms for the Equivalential Calculus: An Application of Automated Theorem Proving to Infinite Domains

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*1 Introduction* Occasionally in mathematics, and especially in various fields of formal logic, there arise various questions of alternate axiomatization. In the equivalential calculus, for example, there are formulas that are known to be strong enough to serve as single axioms. In [15] Peterson gave various possible shortest single axioms for that calculus. With the 10 given there and the one found by Kalman [3], there arose a question concerning the existence of any additional formulas that might also be shortest single axioms. There remained seven formulas yet to be classified in that regard [15].

In this paper we show that each of the four formulas  $XJL$ ,  $XKE$ ,  $XAK$  and  $BXO$  is too weak to be a single axiom. Although the corresponding proof and discussion of the remaining three unclassified formulas are deferred to a later paper, we remark that  $XCB$  is also too weak but both  $XHK$  and  $XHN$  are each "new" shortest single axioms.

The method for obtaining both the results presented here and those that

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