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Cylindrical Decision Problems for System Functions

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1 While investigating the one-one equivalence between various General Combinatorial decision problems, Cleave [1] initiated the concept of system functions. System functions are defined on natural numbers and their values are finite sets of natural numbers. They have many properties in common with those arising from Gödel numbering various combinatorial systems. Thus the decision problems defined for these combinatorial systems can also be defined for system functions. Furthermore, if a property holds for a particular decision problem for all system functions, then it also holds for that decision problem for all these combinatorial systems.

In his study of the one-one equivalence between General Combinatorial decision problems using system functions, Cleave [1] considered only a finite number of decision problems. We have extended his study to include an infinite number of decision problems. This is accomplished firstly, by defining a generalized class of formulas in terms of a first-order language so that each formula in this class corresponds to a decision problem for system functions and secondly, by analyzing these formulas to determine whether the corresponding decision problems for various kinds of system functions are cylinders. As stated by Cleave [1], we take this approach for the following reason: "If P_1 and P_2 are two General Combinatorial decision problems which are many-one equivalent and if each instance of P_1 and P_2 are cylinders, then they are one-one equivalent" ([1], p. 254). This is the best possible equivalence one can obtain.

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