Notre Dame Journal of Formal Logic
Volume 23, Number 4, October 1982

A Note on the Principle of Predication

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Let A be a well-formed formula of first-order modal logic whose only free variable is $\underline{x}$. We shall use the following abbreviations:

$\underline{Mat}(A)$ for $(\underline{x})[\Diamond A \wedge \Diamond\sim A]$
$\underline{Form}(A)$ for $(\underline{x})[\Box A \vee \Box\sim A]$
$\underline{Ban}(A)$ for $((\underline{x})\Box A) \vee ((x)\Box\sim A)$
$\underline{Pred}(A)$ for $\underline{Form}(A) \vee \underline{Mat}(A)$.

We read $\underline{Mat}(A)$, $\underline{Form}(A)$, and $\underline{Ban}(A)$ respectively as: "A is material", "A is formal", and "A is banal"; $Pred(A)$ is the assertion of the Principle of Predication for A.

We prove that if F and M are formulas whose only free variable is $\underline{x}$ such that $\underline{Ban}(F)$, $\underline{Form}(F)$, and $\underline{Mat}(M)$ are true in any suitable T-model, then $\underline{Pred}(F \wedge M)$ and $\underline{Pred}(F \vee M)$ are not acceptable as axioms.

Theorem *The formulas:*

(1) $\sim\underline{Ban}(F) \wedge \underline{Form}(F) \wedge \underline{Mat}(M) \supset \sim\underline{Pred}(M \wedge F)$
(2) $\sim\underline{Ban}(F) \wedge \underline{Form}(F) \wedge \underline{Mat}(M) \supset \sim\underline{Pred}(M \vee F)$

are T-valid.

Proof: Let $\langle W, R, D, Q, V\rangle$ be a T-model ([1], p. 171) and $w_i \in W$. If $V(\sim\underline{Ban}(F) \wedge \underline{Form}(F) \wedge \underline{Mat}(M), w_i) = 1$ then $V(\sim\underline{Ban}(F), w_i) = 1$ and there exist $a, b \in D_i$ such that:

(3) $V^a(\Diamond\sim F, w_i) = 1$ and $V^b(\Diamond F, w_i) = 1$

where V^a and V^b are just like V except for assigning a and b, respectively, to $\underline{x}$.

*Work supported by the G.N.S.A.G.A. of Italian C.N.R.

Received April 10, 1981; revised September 21, 1981