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Automorphisms of ω-Octahedral Graphs

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1 Preliminaries This paper is closely related to [2] which deals with automorphisms of the ω-graph Q_N associated with the ω-cube Q^N and [3] which deals with the ω-graph Oc_N associated with the ω-octahedron Oc^N. We use the notations, terminology, and results of [2]. The propositions of [2] are referred to as A1.1, A1.2, . . ., A2.1, A2.2, . . . etc., those of [3] as B1.1, B1.2, . . ., B2.1, B2.2, . . . etc.

For $n \geqslant 1$ the n-octahedral graph is defined as the complete n-partite graph $K(2, \ldots, 2)$ with two vertices in each of its partite sets ([4], p. 69). Let Oc_n have $\mu = (0, \ldots, 2n - 1)$ as set of vertices and $((0, 1), \ldots, (2n - 2, 2n - 1))$ as class of its partite sets. Define f as the permutation of μ which for $0 \leqslant k \leqslant n - 1$ interchanges $2k$ and $2k + 1$. Call the vertices p and q of Oc_n *opposite,* if they correspond to each other under f, then p and q are adjacent, iff they are not opposite. Throughout this paper the symbols ν, ν_0, ν_1 denote nonempty sets, and μ and μ_ν stand for sets of cardinality $\geqslant 2$. An *involution without fixed points* (abbreviated: iwfp) of a set μ is a permutation f of μ such that $f^2 = i_\mu$ and $f(x) \neq x$, for $x \in \mu$. The iwfp f of μ is an *ω-iwfp,* if it has a partial recursive one-to-one extension. With every iwfp f of μ we associate a graph $G_f = \langle \mu, \theta \rangle$, where θ consists of all numbers $can(x,y) \in [\mu;2]$ such that $f(x) \neq y$. Note that the iwfp f is uniquely determined by G_f. The graph $G = \langle \mu, \theta \rangle$ is *octahedral,* if $G = G_f$, for some iwfp f of μ. The octahedral graph $G_f = \langle \mu, \theta \rangle$ is *ω-octahedral,* if f is an ω-iwfp of μ. The vertices p and q of the octahedral graph G_f are *opposite,* if $f(p) = q$; thus p and q are adjacent iff they are not opposite. According to B2.2 an ω-octahedral graph $G_f = \langle \mu, \theta \rangle$ is a uniform ω-graph for which there exists a nonzero RET N such that $Req\, \mu = 2N$ and $Req\, \theta = 2N(N - 1)$. Define the functions d_0 and d_1 by: $\delta d_0 = \delta d_1 = \varepsilon$, $d_0(x) = 2x$, $d_1(x) = 2x + 1$. With every set ν we associate the sets $\nu_0 = d_0(\nu)$, $\nu_1 = d_1(\nu)$, and $\mu_\nu = \nu_0 \cup \nu_1$. The *standard ω-iwfp associated with the set ν* is the ω-iwfp f of μ_ν such that $f(2x) = 2x + 1$ and $f(2x + 1) = 2x$, for $x \in \nu$. The *standard ω-octahedral graph Oc_ν associated with the set ν* is the ω-graph $G_f = \langle \mu_\nu, \theta_\nu \rangle$,

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