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Nonstandard Propositional Logics and Their Application to Complexity Theory

MICHAEL EVANGELIST*

1 Introduction and background Let Σ^* be the set of all finite-length strings over some fixed alphabet Σ. Then a *language* (over Σ) is a set $L \subseteq \Sigma^*$. Define $P = \{L | L$ is accepted by a deterministic Turing machine (DTM) in a polynomial number of steps$\}$, where the argument to the polynomial function is the length of the input string. *NP* is the analogous family for nondeterministic Turing machines (NDTMs).

The family *P* is widely considered to represent the class of feasibly solvable computational problems. Representative of this class, in a sense to be defined precisely, is the set *S* of satisfiable propositional formulas. Cook [3] has shown that *S* is a member of *P* if, and only if, $P = NP$. (The proof method is similar to that used by Büchi [1] for establishing the unsolvability of the decision problem for the predicate calculus.)

Cook's result has far-reaching implications for the theory of computational complexity, because many interesting combinatorial problems are in the family *NP* but are not known to be in *P*. (See Karp [7]) That is, each of these problems can be solved in polynomial time if, and only if, there is a polynomial time decision procedure for *S*. In addition, as Cook and Reckhow [4] observe, $P = NP$ would also imply an interesting philosophical

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