Notre Dame Journal of Formal Logic
Volume 23, Number 4, October 1982

Axioms for Tense Logic
I. "Since" and "Until"

JOHN P. BURGESS

1 Preliminaries In his thesis [1], H. Kamp enriched tense logic by the addition of two new binary connectives, the *since* operator S and the *until* operator U. Some time afterward, he announced axiomatizability results for the S, U-tense logics of various classes of linear orders. His completeness proofs were (in his own words) "by no means simple", and have never been published, though a manuscript treating certain classes of linear orders is in existence. We will present below axiomatizations for the classes of arbitrary linear orders and of dense and discrete orders, with and without first and last elements. Our completeness proofs, although not entirely trivial, are (relatively) simple modifications of the usual proofs for ordinary tense logic without S and U, using maximal consistent sets.

1.1 Formal syntax We start with a stock of *propositional variables* p_i for $i = 0, 1, 2, \ldots$, writing p, q, r, s for the first few of them. *Formulas* are built up from the p_i using negation ($\sim$), conjunction ($\wedge$), until (U), and since (S). We reserve $\alpha, \beta, \gamma, \delta$ to range over formulas, and A, B, C, D to range over sets of formulas. The *mirror image* of α is the result of replacing each occurrence of U in α by S, and vice versa. In the usual way, inclusive disjunction ($\vee$), material conditional ($\supset$), material biconditional ($\equiv$), constant true ($\top$), and constant false ($\bot$) can be introduced as abbreviations. Further abbreviations, with their suggested readings ('it__the case that') include:

$F\alpha$	for $U(\alpha, \top)$	will be
$P\alpha$	for $S(\alpha, \top)$	was
$G\alpha$	for $\sim F \sim \alpha$	is always going to be
$H\alpha$	for $\sim P \sim \alpha$	has always been
$G'\alpha$	for $U(\top, \alpha)$	is for some time going to be uninterruptedly

Received April 3, 1981