Notre Dame Journal of Formal Logic
Volume 23, Number 4, October 1982

Indenumerability and Substitutional Quantification

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We here establish two theorems which refute a pair of what we believe to be plausible assumptions about differences between objectual and substitutional quantification. We first informally introduce a terminology which enables us to state these assumptions with reasonable clarity. Next we show that these assumptions have actually been made. Finally, in the remaining sections of the paper we prove the refuting theorems and explore the relations between the second and the Skolem submodel theorem.

1 L is any first-order language with countably many names and n-ary predicates as its nonlogical constants. An interpretation I of L is any triple $\langle D, p, d\rangle$ with nonempty set D, assignment p to the n-ary predicates of L of sets of n-tuples of elements of D, and assignment d to the names of L of elements of D. I is countable or indenumerable as D of I is countable or indenumerable. An *interpreted language* is a pair $\langle L, I\rangle$ with I an interpretation of L. $\langle L', I'\rangle$ is an *extension* of $\langle L, I\rangle$ iff L' results from adding countably many names to L and I' is like I except for its assignments to those new names. A definition of truth under an interpretation is *deviant* for interpreted language $\langle L, I\rangle$ iff some existential quantification of L is by that definition not true under I and yet some $x \in D$ of I satisfies its contained formula, and is *irreducibly deviant* for $\langle L, I\rangle$ iff it is deviant for $\langle L, I\rangle$ and for every extension of $\langle L, I\rangle$. I is a *complete* interpretation of L iff I is an interpretation of L and each $x \in D$ of I is by d of I assigned to some name of L. I is a *quantificationally complete* interpretation of L iff I is an interpretation of L, and for each degree-one formula of L, if some $x \in D$ of I satisfies that formula, then some such $x \in D$ of I is by d of I assigned to some name of L.

As propounded in [1], Mates' formal language $\mathcal{L}$ (less its zero-place letters) is one of our L. An interpretation of $\mathcal{L}$ (less the assignment to those

Received January 5, 1981; revised April 20, 1981