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On Certain Normalizable Natural Deduction Formulations of Some Propositional Intermediate Logics

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1 Introduction As was mentioned in [1], the sequence-conclusion approach to natural deduction enables us easily to get natural deduction formulations of some intermediate and modal logics from the corresponding sequent calculi. In this paper, using the method described in [1], pp. 360–366, and the mapping f, defined in the same paper, pp. 371–375, from the class of proofs in a (cut-free) sequent calculus into the class of derivations of the corresponding sequence-conclusion natural deduction system, we will present several normalizable formulations of some intermediate logics.

Our starting point will be some of the known cut-free Gentzen-type formulations of certain intermediate logics.

2 Sequence-conclusion natural deduction First of all, let us say a few words about the sequence-conclusion approach to natural deduction. It is a simple generalization of the well-known Gentzen natural deduction system (see [18]), which was developed by Prawitz (see [11], [12]) and by many subsequent authors (see [10] and [21]), and generalized in different directions, by, e.g., Shoesmith and Smiley [17], Schroeder-Heister [15], Segerberg [16], and so on.

We suppose that the premises and the conclusion of any inference rule are finite sequences of formulas. So, for instance, the rules for the introduction and elimination of implication will be as follows:

$$(\mathrm{I}\!\to)\qquad \frac{\begin{matrix}[A]\\ \Delta, B\end{matrix}}{\Delta, A \to B}$$

$$(\mathrm{E}\!\to)\qquad \frac{\Delta, A \qquad \Lambda, A \to B}{\Delta, \Lambda, B}.$$

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