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A Basic Free Logic

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1 Introduction In [5] Lambert and van Fraassen propose a first-order predicate logic with identity using Fitch's method of subordinate proofs [3]. The system is universally free, that is, valid whether or not the domain is empty and whether or not all terms are assumed to refer to existents. For those cases in which the domain is assumed nonempty, they provide a special rule of vacuous quantifier elimination. With this rule the logic is still free, but not universally.

In [7] I construct a universally free logic with identity, also using Fitch's method, but one simpler and more intuitive than Lambert and van Fraassen's. I begin with a nonfree or "standard" logic, that is, one valid only if all terms are assumed to refer to existents. From this system, I form a free logic valid for only nonempty domains by placing a restriction on just two rules: the rule of universal quantifier elimination and the rule of existential quantifier introduction. This restriction limits nonvacuous universal quantifier elimination and existential quantifier introduction to general subordinate proofs with respect to the instantial term. With a stronger restriction on those same two rules, a universally free logic results. The stronger restriction limits both nonvacuous and vacuous universal quantifier elimination and existential quantifier introduction to general subordinate proofs.

My purposes in this paper are (i) to construct a universally free logic that is simpler and more intuitive than that in [7] and (ii) from it to generate a free logic for only nonempty domains and then a nonfree logic. The universally free system, which I call $S1$, is proposed as "a basic free logic" because of (i) the simplicity of its language and rules and (ii) the intuitively obvious and natural way in which the other two systems develop from it.

The language L of $S1$ is without identity '=' and without the existence symbol '$\exists!$'. The proof technique follows Fitch's with the major exception of a novel

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