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On the Existence of Polynomial Time Algorithms for Interpolation Problems in Propositional Logic

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1 Introduction

1.1 The interpolation problem Let ϕ and ψ be propositional formulas with $\phi(\bar{x},\bar{y}), \psi(\bar{y},\bar{z})$ where $\bar{x},\bar{y},\bar{z}$ are disjoint lists of propositional variables. In other words, the only variables ϕ and ψ have in common are the variables of $\bar{y}$. Let us assume further that $\phi \wedge \psi$ is unsatisfiable. The question is whether there is a propositional formula $\theta(\bar{y})$, built only from the variables shared by ϕ and ψ, such that

(i) $\phi \rightarrow \theta$ and
(ii) $\theta \rightarrow \neg\psi$

are valid.

The interpolation theorem states that this is always the case. We call such a θ the Craig interpolant (in the sequel shortly interpolant) for ϕ and ψ. (Note that we deviate from traditional usage in that one usually calls this the interpolant for ϕ and $\neg\psi$). The interpolation theorem was first stated and proved for first-order logic by Craig [9]. For a discussion of the first-order case without equality the reader is referred to [32]. The statement for propositional logic follows from Craig's theorem trivially. One can also give direct proofs, e.g. by looking at the Boolean function defined by $\Theta = \exists\bar{x}\phi(\bar{x},\bar{y})$. By functional completeness of propositional logic there is a formula $\theta(\bar{y})$ which represents Θ. It is easily seen that θ satisfies (i) and (ii). There is a vast literature about various

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