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Simplicity

VICTOR PAMBUCCIAN

1 Introduction A fairly common practice in today's mathematics is to introduce a theory by 'giving' a first-order language (with a denumerable set of individual variables) $L = L(\mathbf{F},\mathbf{R},r)$ (where $\mathbf{F}$ and $\mathbf{R}$ are two finite collections of symbols, the former called *function*- or *operation*-symbols, the latter *predicate*- or *relation*-symbols; $r: \mathbf{F} \cup \mathbf{R} \to \mathbb{N}$ is a function assigning to each symbol a natural number, its 'arity') and a set Σ of sentences in L. This is done with the aim of introducing special structures for L, called *models.* A *structure* $\mathfrak{A}$ for L has the following components ([55]):

(i) a nonvoid set $u(\mathfrak{A})$, called the universe of $\mathfrak{A}$
(ii) for all $f \in \mathbf{F}$ an $r(f)$-ary function $f_{\mathfrak{A}}: u(\mathfrak{A})^{r(f)} \to u(\mathfrak{A})$
(iii) for all $R \in \mathbf{R}$ an $r(R)$-ary relation $R_{\mathfrak{A}}$ on $u(\mathfrak{A})$, i.e. $R_{\mathfrak{A}} \subseteq u(\mathfrak{A})^{r(R)}$.

A structure $\mathfrak{A}$ for L is called a *model* of a set Σ of sentences in L iff all $\alpha \in \Sigma$ are true in $\mathfrak{A}$ (see [73], [75] for the meaning of 'true').

For example, in *Group Theory*, $\mathcal{G}$, $\mathbf{F} = \{\mu\}$, $\mathbf{R} = \varnothing$, $r(\mu) = 2$, $\Sigma = \{A,B\}$, where

A $\forall xyz\ \mu(x\mu(yz)) = \mu(\mu(xy)z)$
B $\exists e \forall x \exists x'\ \mu(ex) = x \wedge \mu(x'x) = e.$

In most cases Σ is required to be either finite or recursive (see [24] for a definition of this term) and is referred to as an axiom system (AS).

The reasons for accepting these sentences as 'axioms' have been an issue ever since. The conservative view is that of Aristotle, who claims that the axioms must be known by an infallible intuition (*Analytica posteriora* II. 29, 100^b6). From this point of view, these days, both mathematicians and philosophers attack (sometimes, [14], most violently) the formalist point of view, that the AS – as well as the language, i.e. the collections $\mathbf{F}$ and $\mathbf{R}$ – may be freely chosen, subject to the modest requirement of consistency, i.e. noncontradiction. If this sort of criticism comes from an intuitionist, to whom a theory is not constructed within a logical system, but by a creative cognitive process (close to what

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