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Semantical Analysis of Superrelevant Predicate Logics with Quantification

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As is well-known, the definition of the Kripke-type semantics for the relevant propositional logics was given by Routley and Meyer in [9], but the definition of the algebraic semantics of these logics was given by Dunn in [2]. The Kripke-type semantics (called the relevant RPg-spaces) and the algebraic semantics (which we call the simple C_R-matrices) for relevant propositional logics were also defined by Maksimova ([5]). It can be easily proved that Routley and Meyer's semantics are equivalent to relevant RPg-spaces, and that Dunn's semantics are equivalent to simple C_R-matrices. In [5], Maksimova showed that there exists a close relationship between relevant RPg-spaces and simple C_R-matrices. An essential aspect of this relationship is that for any relevant RPg-space there exists the respective simple C_R-matrix, the contents of which are identical with the contents of the relevant RPg-space; and with any simple G_R-matrix it is possible to correlate the respective relevant RPg-space. However the contents of that relevant RPg-space need not be identical with the contents of the initial matrix, though in the case of finite matrices the identity of contents holds.

In this paper we pick up the subject of semantics for quantified relevant logics, which is an important and underdeveloped one. Some basic problems and solutions in this field were noted by Routley in [8]. We introduce here two types of semantics which we call respectively general relevant RPg-spaces (g.r. RPg-spaces) and structurally general relevant RPg-spaces (s.g.r. RPg-spaces). In general, by a g.r. RPg-space we mean any triple $\langle \underline{S}, V, \mathfrak{A} \rangle$ such that $\underline{S}$ is a relevant RPg-space, V is a nonempty set, and $\mathfrak{A}$ is a nondegenerate (V,S)-simple C_{RQ}-matrix, and by an s.g.r. RPg-space we mean any triple $\langle \underline{S}_0, V, \underline{S}_1 \rangle$ such that $\underline{S}_0$ and $\underline{S}_1$ are relevant RPg-spaces and V is a nonempty set. We state that though the contents of any g.r. RPg-space as well as the contents of any s.g.r. RPg-space determine some superrelevant predicate logic, i.e. a predicate logic containing the relevant predicate logic RQ; for the superrelevant predicate logics they are incomplete.

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