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Inconsistent Number Systems

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1 Introduction In a previous paper ([8]), it was shown that there are finite inconsistent arithmetics which are extensions of consistent Peano arithmetic formulated with a base of relevant logic, and also of the set of truths of the classical standard model of arithmetic. In the present paper, the study of the operations of inconsistent number-theoretic structures, especially finite structures, is continued. The interest is particularly in displaying inconsistent theories and associated finite structures which extend standard classical structures, in the sense that all truths of the latter hold also in the former. The principal thesis to be argued on that basis is that classical mathematics is a *special case* of inconsistent mathematics.

The view of mathematics, as based on classical two-valued logic as a deductive tool, has it that from inconsistency all propositions are deducible. Hence, inconsistency-toleration is achieved in the present paper by use of a logic with a weaker deductive relation $\vdash$, the three-valued logic RM3, the third value of which has a natural interpretation, 'both true and false' (cf. Section 2). It should not be thought, however, that theories in which a weaker $\vdash$ is used inevitably lead to sacrifice of some classical propositions. It is one purpose of this paper to demonstrate this, by displaying inconsistent theories which contain various well-known classical consistent complete subtheories.

Aside from its capacity for contradiction containment, RM3 is chosen for two reasons. First, being three valued it is reasonably easy to deal with, particularly in yielding a rich model theory. Second, every RM3-theory displayed is also a theory of all the usual relevant logics such as E and R, which have an independently natural motivation. The interest of those logics for mathematics may be judged accordingly. Indeed, since every classical theory is an RM3-theory and thus also an E- or R-theory, the "special case" thesis above has another dimension: just as consistent mathematics is a special case (under the assumption of consistency or closure under classical deducibility) of inconsistent mathematics, so classical logic is a special case (in which closure under classical deducibility, for instance the rule of Disjunctive Syllogism, holds over a limited

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