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Some Notes on Iterated Forcing With $2^{\aleph_0} > \aleph_2$

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Introduction By Solovay and Tenenbaum ([7]) and Martin and Solovay ([3]) we can iterate c.c.c. forcing with finite support. There have been many works on iterating more general kinds of forcings adding reals (e.g., [4]), getting generalizations of MA, and so on, but we were usually restricted to $2^{\aleph_0} = \aleph_2$. Note only this is a defect per se, but there are statements that we think are independent but which follow from $2^{\aleph_0} \leq \aleph_2$.

Some time ago Groszek and Jech (in [2]) got $2^{\aleph_0} > \aleph_2 + MA$ for a family of forcing wider than c.c.c. but for $\aleph_1$ dense sets only.

In Section 1 we generalize RCS iteration to κ-RS iteration.

In Section 2 we combine from [4], X, XII (i.e., RS iteration and some properness and semicompleteness) with Gitik's definition of order ([1]). (He uses Easton support, each Q $(\{2\}, \kappa_i)$ -complete where for important i, $\kappa_i = i$. His main aim was properties of the club filter on inaccessible: precipitousness and approximation to saturation.)

In Section 3 we get MA-like consequences (strongest-from supercompact). In Section 4 we get that, e.g., for Sacks forcing (though not included), and in the models we naturally get, for every $\aleph_1$ dense subset there is a directed set intersecting all of them.

In Section 5 we solve the second Abraham problem.

The main result was announced (somewhat inaccurately) in [6].

1 On κ-revised support iteration We redo [4], Ch. X, Section 1, with "$< \kappa$" instead countable.

Remarks 1.0:

(1) Now if $P_1 = P_0 * \underset{\sim}{Q}_0$, $\underset{\sim}{q}_1$ a P_1-name, $G_0 \subseteq P_0$ generic over V, then in $V[G_0]$, $\underset{\sim}{q}_1$ can be naturally interpreted as a Q_0-name, called $\underset{\sim}{q}_1/G_0$,

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