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Intensions, Church's Thesis, and the Formalization of Mathematics

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1 Recently a number of authors have proposed including intensional notions, notably epistemic notions, in the underlying logic of mathematics. (See Goodman [6] as well as several of the essays in Shapiro [15]; and for a discussion very close in spirit to the present essay, see Shapiro [14].) These authors have made explicit just what intensional notions are needed for the analysis of mathematical language and how they should be formalized. The present essay is concerned not with those questions, but rather with trying to make the prior case that intensional notions are genuinely needed for the formalization of classical mathematics.

Let us begin with an example of the use of intensional notions in informal mathematical exposition:

> The proof by Gerd Faltings [2] of the Mordell conjecture implies that, for any fixed exponent, there are at most a finite number of counterexamples to the Fermat conjecture for that exponent. That is, let k be some fixed large integer, say $k = 9{,}437{,}512{,}798$. Then there is some finite integer n such that there are exactly n pairs of rational numbers p and q such that
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> $$p^k + q^k = 1.$$
>
> If the Fermat conjecture for k is true, then $n = 0$. At the moment, however, the number n is not known. In fact, we do not even know how to bound n. Nevertheless, there is some hope that bounds may be obtained. For the first time, we seem to be in a position to make systematic progress on this 350-year-old problem.

It is a remarkable fact that the main logical tradition, stemming from Frege, holds that the last four sentences of the above discussion are not part of

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