A Simplified Natural Deduction Approach
to Certain Modal Systems

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The natural deduction approach to standard logic has a number of virtues, among them being ease of application. However, attempts in the literature to combine natural deduction with modality have generally resulted in cumbersome systems little more flexible than straightforward axiomatics.[1] This paper presents a natural deduction approach to the T-B-$S4$-$S5$ ring which, if I am not mistaken, renders them as simple as the ordinary first-order logic taught in introductory courses. One of its most important advantages is semantic transparency: it virtually wears its interpretation on its sleeve. It will perhaps become obvious that this approach has it roots in the metaphor of modal logic as quantification over possible worlds, though this relationship will not be spelled out in any detail.

1 Intuitive groundwork   Think of possible worlds as flagged or represented by distinct numerals, with 1 representing the actual world and other numerals representing others. In modal logic, truth-predicates are not applied absolutely, but only relative to particular possible worlds. We may observe this graphically by indexing each sentence to a world at which it is (asserted to be) true, by attaching that world's numeral to the propositional expression as a subscript. P_1 will mean that P (is true) at world 1; $(Q \& R)_2$ will mean that $(Q \& R)$ at world 2, and so on.

Modal systems differ in their construal of the access relation between worlds. Let possible-world indices be arranged in strings or *chains*. Then the access relationships between worlds represented by those indices may be mapped or represented by spatial relationships between numerals in the chain.