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Relatively Diophantine Correct Models of Arithmetic

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A model M of Peano arithmetic is called *diophantine correct* if every polynomial which has a root in M already has a root in $\mathbf{N}$ (the standard natural numbers). Lipshitz [2] has shown that if M is a countable nonstandard model of Peano arithmetic, then M is diophantine correct if and only if for every nonstandard $\alpha \in M$ there is an embedding of M onto an initial segment of $[0, \alpha] = \{x \in M | x \leq \alpha\}$. In this paper we extend this result to the case of a countable model M being diophantine correct relative to a submodel N (see definition in Section 1 below).[1]

1 We are repeatedly going to talk about N-polynomials $p(\bar{x})$, where N is a countable model of Peano arithmetic, and about the result of substituting a sequence of elements $\bar{a}$ (usually in a larger model M) into $p(\bar{x})$, getting $p(\bar{a})$. By an N-polynomial $p(\bar{x})$ we mean a nonstandard polynomial which is coded by its Gödel-number $\ulcorner p(\bar{x}) \urcorner$. Notice that N-polynomials will, in general, have a nonstandard number of variables, as well as nonstandard sums, products, and coefficients. When phrases such as "the polynomial $p(\bar{x})$" or "$p(\bar{a}) = b$" appear in formulas the reader is to understand that the formula actually is one involving the Gödel numbers of such polynomials. We shall repeatedly use the fact that the sets of Gödel numbers of polynomials, and of formulas "$p(\bar{x}) = y$", are defined by Σ_1 formulas and, using the results of Matijasevič [3], by π_1 formulas. We will assume the reader is familiar with the basic model theory and coding techniques used in the study of nonstandard models of arithmetic (see, e.g., Pillay [5]).

Let M and N be models of Peno arithmetic. M is *N-diophantine correct* if for every N-polynomial $p(\bar{x})$, if $p(\bar{x})$ has a zero in M then it already has one in N.

Wilkie [6] has shown that every countable model N of Peano arithmetic has an end extension M such that $N \simeq M$ and such that M solves a diophantine equation with coefficients in N that is not solvable in N. Hence, every countable non-

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