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Decision Procedure for a Class of $(L_{\omega_1\omega})_t$-Types of T_3 Spaces

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The $(L_{\omega\omega})_t$-types of T_3 spaces are introduced in [1]. An effective procedure is then obtained to decide whether a type is satisfiable in some T_3 space. The expressibility of $(L_{\omega_1\omega})_t$ for T_3 spaces is studied in [2]. For this purpose a class of $(L_{\omega_1\omega})_t$-types is introduced and in this way we obtain a characterization of the $(L_{\omega_1\omega})_t$-equivalence for a wide class of T_3 spaces. In the present paper, we prove that there is a decision procedure for this class of types.

1 Preliminaries Suppose that A is a T_3 space and A^* is a subset of A. The *n-move game* $G_n(A^*, A)$ between two players, I and II, is defined as follows. In his i-th move ($i = 1 \ldots n$) player I chooses an arbitrary finite sequence $a_1, \ldots, a_r$ of points in A and then in his i-th move player II chooses a sequence of r neighborhoods U_1 of $a_1, \ldots, U_r$ of a_r in A. Let $U_1', \ldots, U_m'$ be all the neighborhoods chosen by II during the game. Player I wins if $A^* \subset U_1' \cup \ldots \cup U_m'$; otherwise, player II wins. Then, A^* is *accessible* (in the space A) if for some $n \in \omega$ player I has a winning strategy in the game $G_n(A^*, A)$. With this notion we can study the behavior of convergence. If $a \in A$ we say that A^* converges to a, $A^* \to a$, if a is an accumulation point of A^*. If $A^* \to a$ the following two types of convergence are considered:

(i) $A^* \xrightarrow{0} a$, if for every neighborhood U of a we have that $A^* \cap U$ is not accessible.
(ii) $A^* \xrightarrow{1} a$, if there is a neighborhood U of a with $A^* \cap U$ accessible.

The set S_n of n-types is then defined by induction on n:

$$S_0 = \{*\},\ S_{n+1} = P\left(\bigcup_{\lambda=0,1} \{(\alpha, \lambda) : \alpha \in S_n\}\right),$$

where $P(X)$ denotes the power set of X.

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