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Maximal p-Subgroups and the Axiom of Choice

PAUL E. HOWARD and MARY YORKE*

According to Sylow's well-known theorem, if p is a prime any finite group G has a Sylow p-subgroup, that is, a subgroup of order p^k where p^k is the highest power of p which divides the order of G.

The notion of Sylow p-subgroups has been generalized to infinite groups (see, for example, [5], p. 58; [2], Sections 54 and 85; and [6], Chapter 6) by the following:

Definition *A* Sylow p-subgroup of G is a maximal p-subgroup of G.

With this definition, the generalization of the Sylow theorem (ST) to infinite groups, i.e.,

ST *If p is a prime, every group has a Sylow p-subgroup*

is an easy consequence of Zorn's lemma.

We show in Section 2 that ST is actually equivalent to Zorn's lemma by showing ST implies the axiom of choice.

Section 3 contains a weakened version of ST, and its relationship to the axiom of choice for sets of finite sets is studied.

1 Definitions and preliminary results We will follow the usual convention of denoting a group $(G, \circ)$ by G when the choice of notation for the operation on the group does not concern us. If y is a set, we will denote by S_y the symmetric group on y. If $\sigma, \tau \in S_y$, $\sigma \circ \tau$ is the permutation defined by $(\sigma \circ \tau)(t) = \sigma(\tau(t))$.

If $t_1, t_2, \ldots, t_n \in y$, $(t_1; \ldots; t_n)$ denotes the cycle σ defined by

$$\sigma(t_i) = \begin{cases} t_{i+1} & \text{if } 1 \leq i < n \\ t_1 & \text{if } i = n, \end{cases}$$

and $\sigma(t) = t$ otherwise.

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