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Cantor-Bendixson Spectra of ω-Stable Theories

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1 Introduction In the following, we shall mean by theory a first-order, countable, complete, quantifier-eliminable theory.

The idea of classifying ω-stable theories by the analysis of the Boolean algebras of the definable subsets of their countable models arises from [3] and is based on the remark that a theory T is ω-stable if and only if, for every countable model M of T, the Boolean algebra $B(M)$ of the definable subsets of M is superatomic. In fact, it is well-known that, for every Boolean algebra B, an ascending chain $\{I_\nu(B)\colon \nu \text{ ordinal}\}$ of ideals of B can be defined in this way:

1. $I_0(B) = \{0\}$
2. $I_1(B)$ is the ideal of finite elements of B
3. for every ordinal ν, $I_{\nu+1}(B)$ is the preimage in B of $I_1(B/_{I_\nu(B)})$ in the canonical homomorphism of B onto $B/_{I_\nu(B)}$
4. for every limit ordinal λ, $I_\lambda(B) = \bigcup\limits_{\nu<\lambda} I_\nu(B)$.

In particular, when B is superatomic, there is an ordinal μ such that $I_\mu(B) = B$; let μ be the least ordinal with this property, then μ is a successor ordinal, and we may define:

α_B = predecessor of μ = least ordinal ν such that $I_\nu(B) \neq B$
d_B = number of atoms in $B/_{I_{\alpha_B}(B)}$.

We have the following:

(i) $\alpha_B < \omega_1$ if B is countable
(ii) $d_B < \omega$
(iii) for every ordered pair (α, d) with $1 \leq \alpha < \omega_1$, $1 \leq d < \omega$, there is a countable superatomic Boolean algebra B such that $(\alpha, d) = (\alpha_B, d_B)$

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