Invertible Definitions

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Introduction   A concept of informational equivalence between relations is explicated to generalize some suggestions by Geach. It is shown that two relations are informationally equivalent if and only if each can be defined in terms of the other without the use of quantifiers. It is shown that there is a general method for listing the j-place relations informationally equivalent to an arbitrary given i-place relation if and only if $i \le j$. The equivalence classes of the relation of informational equivalence are characterized as the invariants of the group of invertible quantifier-free definitions, for $i = j$.

Quantifier-free definition is contrasted with general first-order definition by means of an example of two first-order interdefinable relations which are not interdefined by any pair of mutually inverse first-order definitions.

1   Geach ([3], pp. xi–xii, 25–26, 33, 52) has suggested that one property or relation may be so closely connected to another that to have the concept of either is to have the concept of both. For example, to know what is *red* is to know what is *not red*, and to know what is *to the left of* what is to know what is *to the right of* what. If a relation is the same as its contradictory and its converse in this respect, it is presumably also, by transitivity, the same as the contradictory of its converse in the same respect. Since Geach claims to refute some theories of concept acquisition and possession on the basis of their neglect of this equivalence relation, a formal characterization of it would seem desirable, so that a general principle covering his remarks can be stated and tested.

Geach's suggestion is plausible if concepts are thought of as discriminative capacities. For vividness, imagine an ω-sequence of labeled objects to which we have no direct access, but about which we can gather information via a robot (the discriminative capacity). Suppose that we can use the robot to discover whether the object with a given label is red (when the instruction 'Red' is input, followed by the label, the robot moves along the line until it finds the object with

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