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# On Defining Sentential Connectives

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*1 Introduction* Since Lesniewski[1] first set the rules of definition for the introduction of new notation into a logical system, sentential connectives have been almost completely neglected. Traditional accounts of the rules of definition, like Carnap's ([2], pp. 66–73) and Suppes' ([9], pp. 154–162), treat only relation symbols, operation symbols, and individual constants. Even when not neglected, as by Tarski ([10], pp. 150–151), the rule for defining connectives covers their introduction in but a few expressively powerful languages. In this paper, then, we develop a sufficiently general, purely syntactical rule for defining sentential connectives. And because traditional accounts of the rules of definition usually call for us to frame definitions as material equivalences within the object language, these treatments cannot provide for the introduction of new notation into weaker languages which lack a suitable equivalence connective. So, we also extend our treatment of the definition of connectives to other parts of speech, thus allowing the introduction of new symbols into impoverished systems as well.

To begin, recall the rule which Patrick Suppes offers for defining relation symbols:

> An equivalence $D$ introducing a new $n$-place relation symbol $P$ is a proper definition in a theory if and only if $D$ is of the form
>
> $$P(v_1, \ldots, v_n) \equiv S,$$
>
> and the following restrictions are satisfied: (i) $v_1, \ldots, v_n$ are distinct variables, (ii) $S$ has no free variables other than $v_1, \ldots, v_n$, and (iii) $S$ is a formula in which the only non-logical constants are primitive symbols and previously defined symbols of the theory. [9], p. 156

Because we find analogous concepts later, we call these three restrictions: (i) *n-arity* (since we require that $P$ take $n$ distinct variables), (ii) *parametric relevance*

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