

## On the Brink of a Paradox?

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In his introduction to the translation of Frege's *Begriffsschrift* published in *From Frege to Gödel: A Source Book in Mathematical Logic*, Jean van Heijenoort writes ([5], p. 3):

. . . If we also observe that in the derivation of formula (91) he substitutes  $\mathfrak{F}$  for  $f$ , we see that [Frege] is on the brink of a paradox.

What this means may, at the moment, be obscure, but it is my aim to illuminate this passage and as I continue its meaning will become clear. van Heijenoort's claim that Frege is "on the brink of a paradox" is, of course, metaphorical, for Frege's system either leads to a paradox or it does not. Terrell Bynum, in his edition of the *Begriffsschrift*, maintains that no paradox can be generated in Frege's system. He writes ([1], p. 182):

Van Heijenoort is in error in supposing that any paradox can arise from the deductive procedure Frege uses here.

In this paper I attempt to resolve this dispute. In the first section I reconstruct the system of Frege's *Begriffsschrift* and show that the reconstructed system is equivalent to a standard second-order predicate calculus, and then demonstrate the consistency of the reconstructed system.<sup>1</sup> I conclude, then, that if van Heijenoort is claiming that the system leads to a paradox or inconsistency, the dispute is settled on the side of Bynum. In the second part I consider the interpretation of the system of the *Begriffsschrift*. Frege is not clear about how the system is to be interpreted. In light of Frege's later writings on the distinction between function and object, the interpretation of second-order quantifications presents some difficulties for Frege. These difficulties may be seen as an anticipation of the well-known problem with the concept *horse*. The nature of

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