Semantical Antinomies in the Logic of Sense and Denotation

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0 Informal introduction   Frege's semantical ideas about sense and denotation have very often been discussed as if the main problem were to work out a satisfactory *intensional semantics* for the natural languages. Frege himself was always content to discuss matters of intensionality in connection with unformalized natural languages.[1] But given his very Platonistic views, it seems clear that he did not regard this as getting to the heart of the matter. From that perspective it is arguable that the main thing that is needed is an adequate *intensional logic* or (what may not be quite the same thing) a *theory of intensional entities*. Once this is in hand, it would seem to be a relatively simple matter to use the theory to construct an intensional semantics as desired. It was presumably with some such idea in mind that Alonzo Church constructed his Logic of Sense and Denotation ([5], [7], [8]). The name is actually somewhat misleading (as Wells notes in [14]): the Logic of Sense and Denotation is a theory of the entities *suitable* to be senses of this or that expression in this or that actual or possible language—i.e., it is a theory of *concepts* (in Church's sense, not Frege's). No language nor class of languages is singled out for discussion and no *intensional semantics* is constructed with some of the concepts *as* senses and various things *as* denotations.[2] The main concern is with the intensional entities and their relations to one another and to other things (especially the *concept relation*, which holds between a concept and the entity, if any, falling under it).

Unfortunately, when the actual details are supplied, contradictions threaten to appear. Some of these are analogous to the Epimenides Paradox in that they require additional, empirical but clearly possible, premises. Others are analogous to the Zermelo-König Antinomy and produce contradictions only in the presence of the Axiom of Choice.[3] Still others are intrinsic to the Logic of Sense and Denotation (without the Axiom of Choice) but turn on the adoption of a "criterion of identity" for concepts of a stringent, but quite plausible, sort.

A satisfactory completion of the Frege–Church project requires some reso-