

A Model-Theoretic Reconstruction of Frege's Permutation Argument

PETER SCHROEDER-HEISTER*

1 Introduction In Section 10 of [3] (p. 17¹) Frege claims:

. . . without contradicting our setting ' $\epsilon\Phi(\epsilon) = \epsilon\Psi(\epsilon)$ ' equal to ' $\mathcal{U}\Phi(a) = \Psi(a)$ ' it is always possible to stipulate that an arbitrary course-of-values is to be the True and another the False.

In what follows this assertion will be called the *identifiability thesis* since it states that two arbitrary but different courses-of-values can be identified with the truth-values. Frege considers the identifiability thesis a consequence of his previous argumentation ([3], p. 17, lines 23–36)² which, following Dummett ([1], p. 408), will be called the *permutation argument*, because the concept of a one-one mapping from the considered domain of objects onto itself, i.e. a permutation, is essential for it. More precisely, Frege gives a specific permutation which interchanges the True and the False with two objects denoted by names of the form ' $\eta\Phi(\eta)$ '. This paper attempts to show that the permutation argument is correct, but that it is no argument *for* the identifiability thesis, and that the same holds for related arguments using arbitrary transformations of the domain of objects into itself instead of permutations. This contradicts every interpretation of Section 10 of the "Basic Laws" with which I am familiar, even the most careful and detailed presentation by Thiel [6].³ The validity of the identifiability thesis itself and the conclusions which can be drawn from it⁴ are of course quite independent of this result. However, at the end a counterexample will be given which

*This work was partly stimulated by discussions with the participants of a seminar on Frege's "Basic Laws of Arithmetic" held in winter 1981/82 at the University of Konstanz, especially with Franz-Viktor Kuhlmann and Pirmin Stekeler-Weithofer. An earlier version of the manuscript was written during a stay at the Institute for Advanced Studies in the Humanities of the University of Edinburgh which was financially supported by the British Council. The results have been presented in part at the Second Frege Conference, Schwerin, September 10–14, 1984. I should like to thank an anonymous referee for helpful suggestions and Stella Lewis for checking the English.