Frege's Permutation Argument

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At the beginning of section 10 of his *Grundgesetze*, Frege complains that the stipulation embodied in his abstraction principle[1] "by no means fixes completely the denotation of a name like '$\acute{\epsilon}\Phi(\epsilon)$' " ([1], p. 46), and he proceeds to give a technical argument–which, following Dummett, we shall call the permutation argument–in support of this claim. He writes:

> If we assume that $X(\xi)$ is a function that never takes on the same value for different arguments, then for objects whose names are of the form '$X(\acute{\epsilon}\Phi(\epsilon))$' just the same distinguishing mark for recognition holds, as for objects signs for which are of the form '$\acute{\epsilon}\Phi(\epsilon)$'. To wit, '$X(\acute{\epsilon}\Phi(\epsilon)) = X(\acute{\alpha}\Psi(\alpha))$' then also has the same denotation as '$\text{--}\!\!\!\overset{\alpha}{}\text{--}\Phi(\mathfrak{a}) = \Psi(\mathfrak{a})$'. From this it follows that by identifying the denotation of '$\acute{\epsilon}\Phi(\epsilon) = \acute{\alpha}\Psi(\alpha)$' with that of '$\text{--}\!\!\!\overset{\alpha}{}\text{--}\Phi(\mathfrak{a}) = \Psi(\mathfrak{a})$', we have by no means fully determined the denotation of a name like '$\acute{\epsilon}\Phi(\epsilon)$'—at least if there does exist such a function $X(\xi)$ whose value for a value-range as argument is not always the same as the value-range itself.[2]

Later in section 10 Frege, appealing to a variant of the permutation argument, argues that "it is always possible to stipulate that an arbitrary value-range is to be the True and another the False" (p. 48). In a challenging article full of interesting observations [3], Peter Schroeder-Heister claims this argument is fallacious.[3] According to Schroeder-Heister, the identifiability thesis (the thesis that it is always possible to stipulate that an arbitrary value-range is to be the True and another the False) cannot be established in the way attempted by Frege and is, in any case, false. We hope to show that Schroeder-Heister's model-theoretic reconstruction of Frege's argument misrepresents the argument and (in particular) its conclusion, the identifiability thesis. We believe that, correctly construed, the argument constitutes a perfectly sound demonstration of the identifiability thesis.[4]

In fact, Schroeder-Heister considers several ways in which Frege's argument can be translated into a model-theoretic framework. In each case he shows that