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Near Coherence of Filters, I: Cofinal Equivalence of Models of Arithmetic

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Abstract We define cofinal equivalence to be the smallest equivalence relation on models of arithmetic such that every model is equivalent to all of its cofinal submodels. It is easy to classify tall models (those with no last sky) up to cofinal equivalence, but an attempt to do the same for short models leads to questions independent of the axioms of set theory. We introduce the set-theoretic principle (NCF) of near coherence of filters, whose effect is to make all short nonstandard models of arithmetic cofinally equivalent. We give several equivalent formulations and several consequences of NCF.

1 Introduction and preliminaries We are primarily concerned with models of full arithmetic, that is, with elementary extensions of the standard model whose universe is ω and whose relations and functions are all of the (finitary) relations and functions on ω. Many of the results in the early part of the paper remain true for arithmetic formulated in smaller languages, but we leave this extension to the reader.

The classification, up to isomorphism, of models of arithmetic appears hopelessly complicated, so it is reasonable to attempt a classification up to some coarser equivalence relation. We introduce in this paper one such equivalence relation, called cofinal equivalence, for which a reasonable classification may be possible.

The concept of cofinal equivalence is based on ignoring the changes that a model undergoes when new elements are added below elements already present. More precisely, a model is cofinally equivalent to each (isomorph) of its cofinal submodels, and cofinal equivalence is the smallest equivalence relation with this property.

It turns out (see Lemma 3 below) that the cofinal-equivalence classes are of two sorts, those consisting of tall models and those consisting of short models.

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