Notre Dame Journal of Formal Logic
Volume 27, Number 4, October 1986

A New Variant of the Gödel-Malcev Theorem for the Classical Propositional Calculus and Correction to My Paper—The Connective of Necessity of Modal Logic S_5 is Metalogical

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The Gödel-Malcev theorem says that every consistent system has a model. The theorem can be formulated as follows:

(1) If X is a consistent system then there exists such a $0-1$ valuation h that $hX \subseteq \{1\}$.

It has the following syntactical version:

(2) If X is a consistent system then there exists such a substitution s that $sX \subseteq CPC$.[1]
(CPC = the set of all classical theses.)

Our variant is a strengthening of (2) and has the following form:

(3) If X is a consistent system then there exists such a substitution s that $X = s^{-1}CPC$.

This theorem can be treated as a *theorem on representation of consistent systems by substitutions.*

Let $Ln = (Fr, \neg, \wedge, \vee, \rightarrow, \equiv)$ be an algebra of formulas formed by means of propositional variables $p_0, p_1, \ldots$ and the connectives: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication), and $\equiv$ (equivalence). By the symbol $Fr^{(k)}$ ($k = 1, 2, \ldots$) we denote the set of all formulas formed by means of variables $p_0, \ldots, p_{k-1}$ (and connectives), and by the symbol $Ln^{(k)}$ we denote the subalgebra $(Fr^{(k)}, \neg, \wedge, \vee, \rightarrow, \equiv)$ of algebra Ln determined by variables $p_0, \ldots, p_{k-1}$. By the symbol Cn we denote an operation of consequence (consequence for short) determined by the classical theses, and the rule of modus ponens defined on all subsets of the set Fr (i.e., $Cn(X) = \bigcap \{Y \subseteq Fr$: $X \cup CPC \subseteq Y$ and the set Y is closed under the rule of modus ponens$\}$ $(X \subseteq Fr)$).

Received May 9, 1984